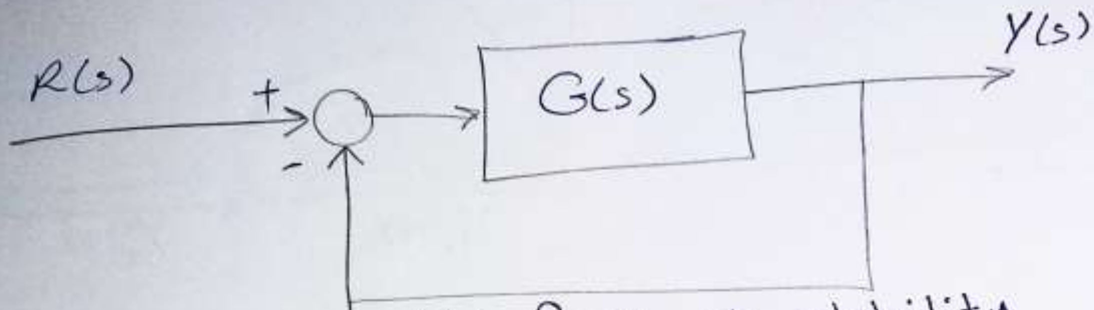
Lec 14 نظم وإشارات

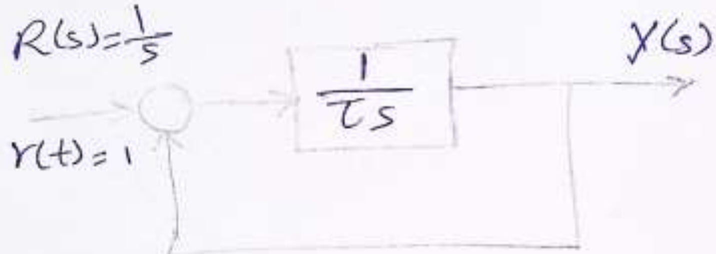
* stability of Control systems :-



unity feedback stability

* unit step response:-

1 st order system:-



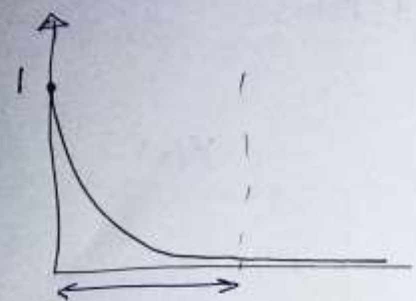
$$\frac{Y(s)}{R(s)} = \frac{1}{1+\tau s} = \frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}$$

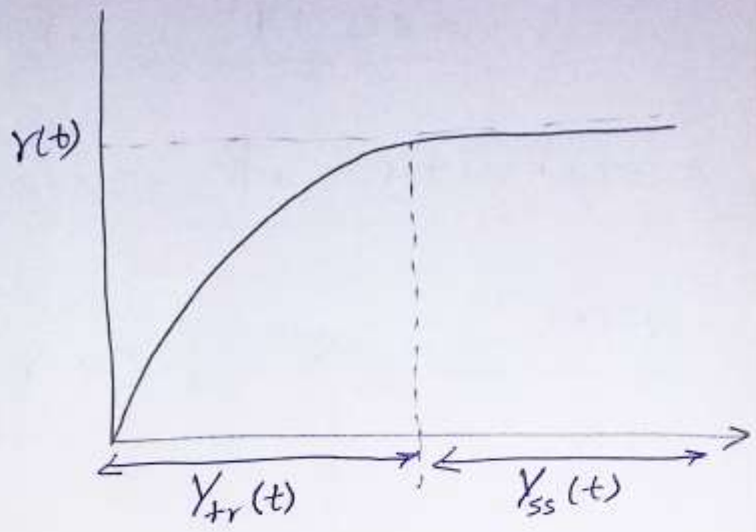
$$Y(s) = \left(\frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}\right) R(s) = \left[\frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}\right] * \frac{1}{s}$$

$$y(t) = 1 - e^{-t/\tau} = Y_{ss}(t) + Y_{tr}(t)$$

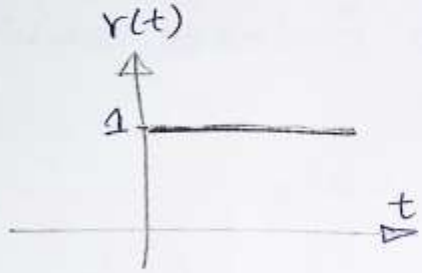
$$Y(s) = \frac{1}{s} \cdot \frac{-1}{s+\frac{1}{\tau}}$$

$$Y_{tr}(t) = e^{-t/\tau}$$



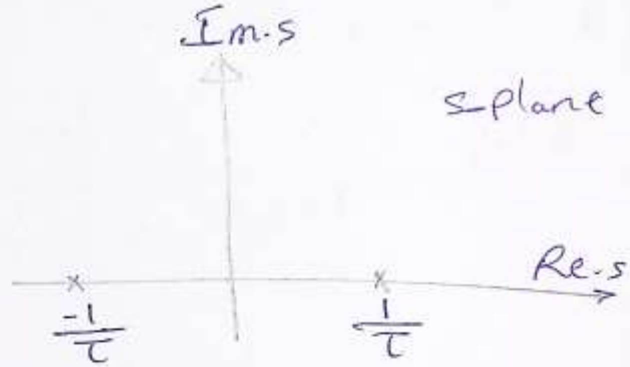


bounded input



stable system

Poles $\frac{-1}{\tau}$



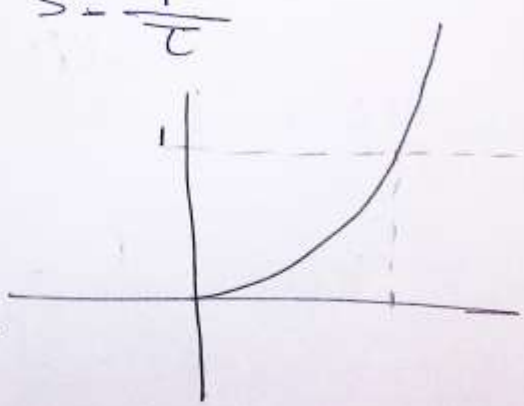
$$\frac{X(s)}{R(s)} = \frac{1}{1-\tau s}$$

$$Y(s) = \frac{\frac{-1}{\tau}}{s - \frac{1}{\tau}}\left(\frac{1}{s}\right) = \frac{1}{s} \cdot \frac{-1}{s - \frac{1}{\tau}}$$

$$y(t) = 1 - e^{-t/\tau}$$

unbounded

unstable



* From unity feed back stability:-

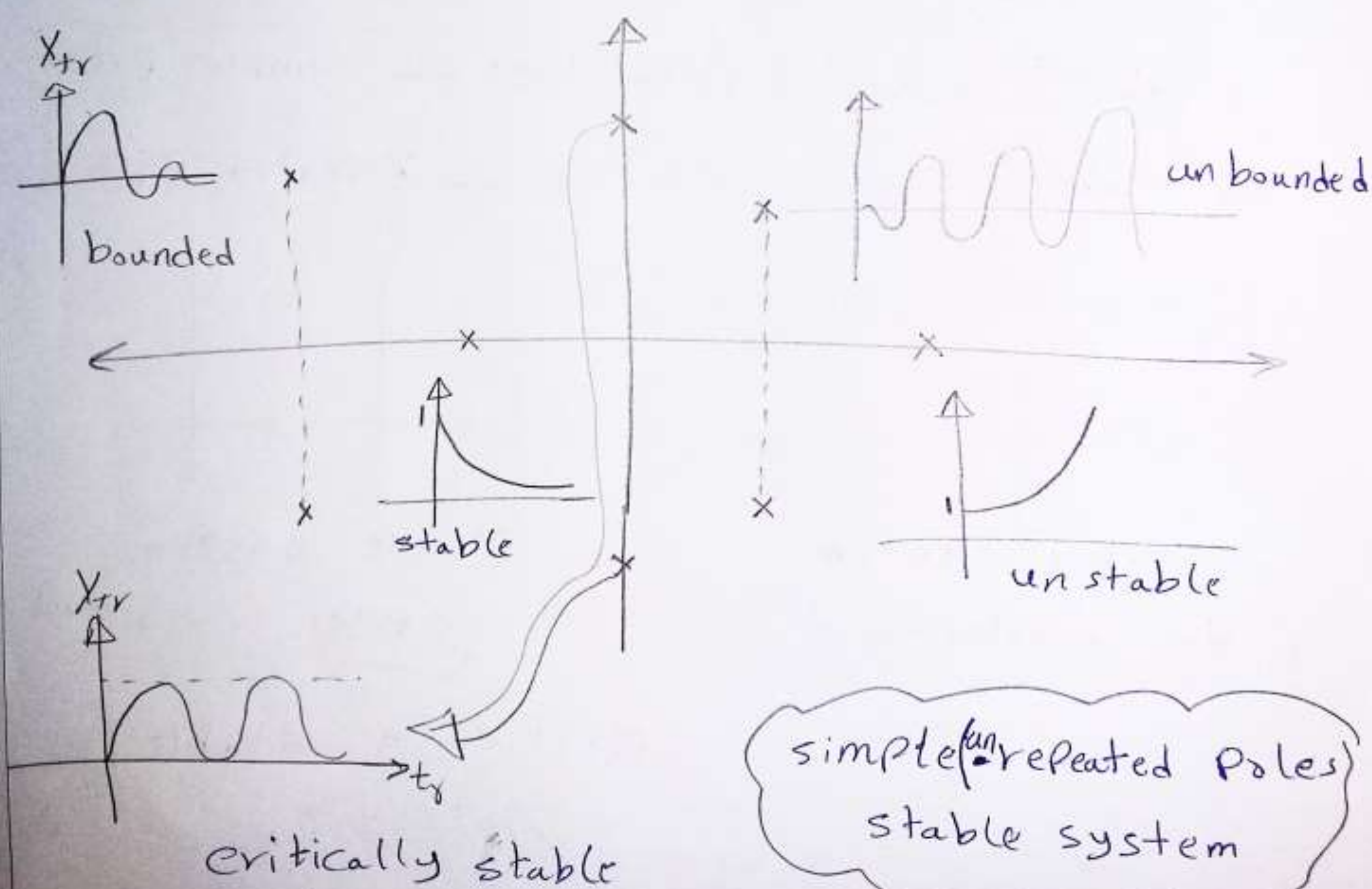
→ For n^{th} order system:-

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{N(s)}{D(s)}$$

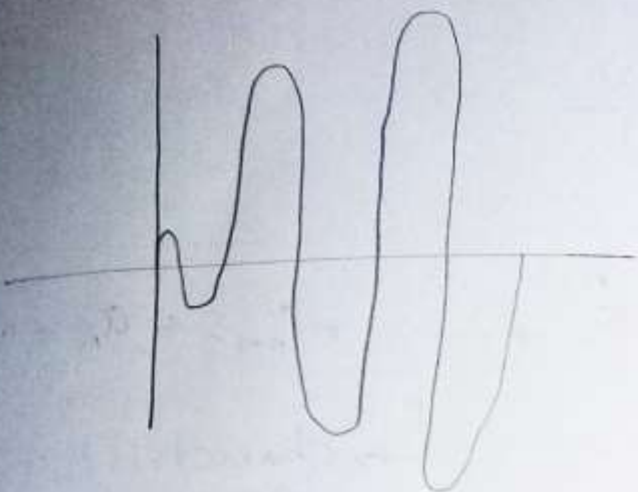
$$\boxed{D(s) = 1 + G(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0}$$

↳ charactristic eqn.

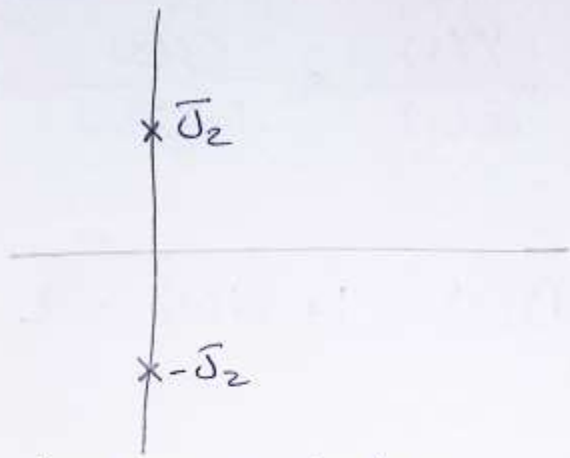
$$y(t) = Y_{ss}(t) + X_{tr}(t)$$



ـــ "Poles" على المحور الرأسي تكون إما "simple"
أو "repeated"

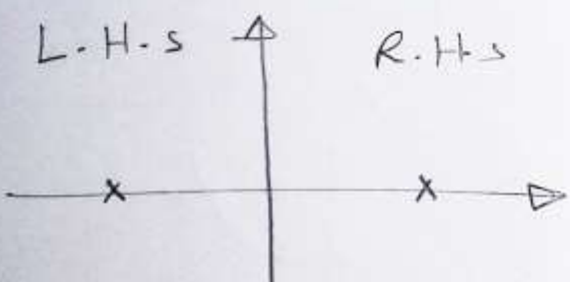


unstable system

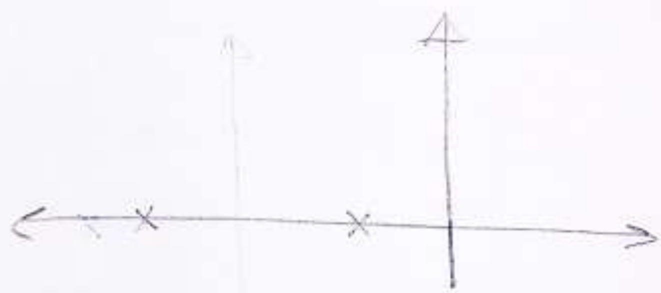


repeated Poles

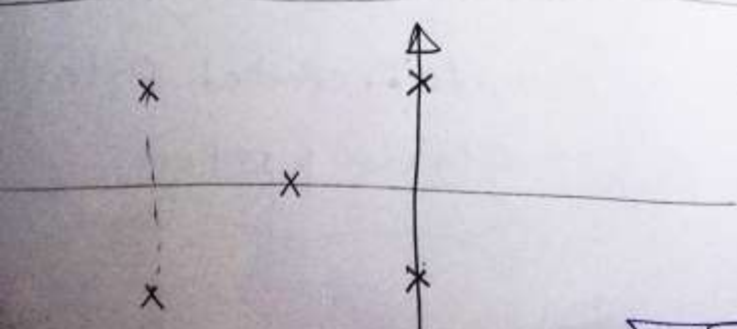
"simple Pole" ← يجعل "system" يكون "stable".
"repeated Pole" ← " " " "unstable"



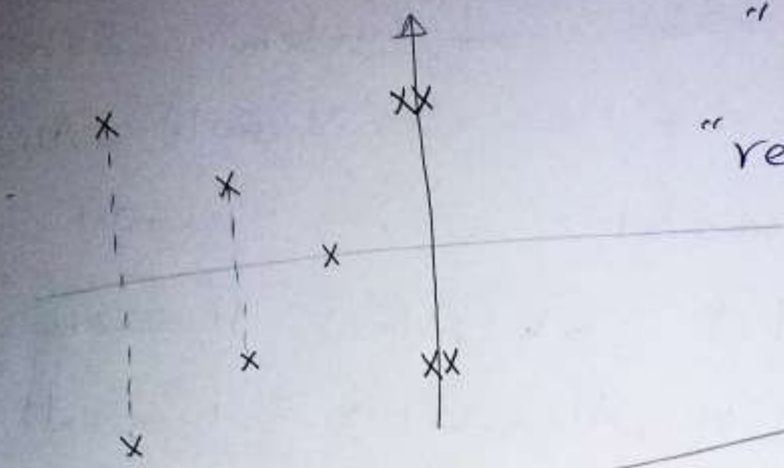
unstable system
لوجود "Poles" في R.H.s



Stable system
لعدم وجود "Poles" في



Critically stable
لوجود "Poles" على المحور الرأسي



"unstable system"

"repeated poles" لوجود

تحدد ال (system) هل (stable) أو (unstable) بدون حساب الجذور.

* Routh-Harwitz array

$$
\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \\
 & a_1 & a_3 & a_5 \\
 & b_1 & b_2 & b_3 \\
 & c_1 & c_2 & c_3 \\
 & \vdots & & \\
s^1 & \vdots & & \\
s^0 & a_n & &
\end{array}
$$

chls eqn معاملات

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

← يكون "system" ← stable اذا كان عناصر العمود الأول لها نفس الإشارة سواء + أو - ولو فيه اختلاف في الإشارة يكون unstable.

← عدد الـ (Poles) ناحية (R.H.s) يساوي عدد المتغيرات في إشارة العمود الأول.

Examples $s^5 + 8s^4 + 2s^3 + 4s^2 + 2s + 4 = 0$

s^5	1	2	2	$\Rightarrow$	+1	2	2
s^4	8	4	4	$\xRightarrow{\div 4}$	+2	1	1
s^3	$\frac{3}{2}$	$\frac{3}{2}$		$\Rightarrow$	+1	1	
s^2	-1	1		$\Rightarrow$	- -1	1	
s^1	2			$\Rightarrow$	+2		
s^0	4			$\Rightarrow$	+4		

$\Rightarrow$ There are two sign changes in the elements of the 1st Column.

$\therefore$ The system is unstable:-

3 L.H.s $\Leftarrow$ 2 R.H.s $\leftarrow$ 5 Pole $\leftarrow$

Example $s^4 + s^3 + s^2 + s + 3 = 0$

sol

نستبدل 0 ← ϵ
لأنه لا يجب القسمة على 0 .

$$
\begin{array}{c|ccc}
s^4 + & 1 & 1 & 3 \\
s^3 + & 1 & 1 & \\
s^2 + & \cancel{0}\ \epsilon & 3 & \\
s^1 - & \frac{\epsilon - 3}{\epsilon} & & \\
s^0 + & 3 & &
\end{array}
$$

$\longrightarrow$ لأن ϵ صغيرة جداً

← (unstable) لتغير الإشارة.

$$\underbrace{\text{2 in R.H.S} \qquad \text{2 in L.H.S}}_{\text{4 poles}}$$

← إذا وجدنا أن عناصر الصف جميعها = صفر نحسب

$A(s)$ ثم نأتي بتفاضل $\frac{dA(s)}{ds}$

$$
\begin{array}{|ccc}
x & x & x \\
x & x & x \\
0 & 0 & 0 \\
x & x & x
\end{array}
\quad \longrightarrow A(s)
$$

$A(s)$ من الصف الذي يسبق الصف
الصف عناصره = صفر.

Example $s^4 + s^3 - s - 1 = 0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 0 & -1 \\
s^3 & 1 & -1 & \\
s^2 & 1 & -1 & \Longrightarrow A(s) \\
s^1 & 0 & \longrightarrow \text{all zero row} & \\
s^0 & -1 & &
\end{array}
$$

auxillary eqn $A(s) = s^2 - 1 \Rightarrow \frac{dA(s)}{ds} = 2s$

- يوجد 1 Pole R.H.S لأن الإشارة تغيرت مرة واحدة.

- الصف اللى كله أصفار لابد أن يكون صف فردى.

Routh array

No all zero row

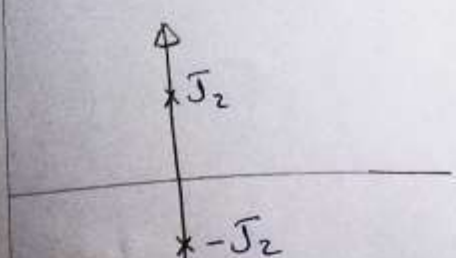
$$\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{cases} \text{stable} \\ \text{unstable} \end{cases}$$

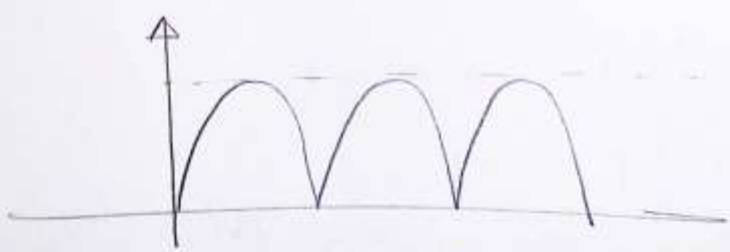
all zero row

$$\begin{vmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \\ x & x & x \end{vmatrix} \begin{cases} \text{critically stable} \\ \text{unstable} \end{cases}$$

$$\left.\begin{matrix} & x & x & x \\ A_1(s) \rightarrow & x & x & x \\ & 0 & 0 & 0 \\ A_2(s) \rightarrow & x & x & x \\ & 0 & 0 & 0 \end{matrix}\right\} \text{unstable system}$$

$A(s) = 0$





sustaned (constant) oscillations

$s = Re.s + Im.s$
$= \sigma + j\omega$

Freq. of oscillations $= \omega$